Phenomenological Implications of a Class of Neutrino Mass Matrices

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Abstract

The generic predictions of two-texture zero neutrino mass matrices of class A in the flavor basis have been reexamined especially in relation to the degeneracy between mass matrices of types A_1 and A_2 and interesting constraints on the neutrino parameters have been obtained. It is shown that the deviation of the atmospheric mixing angle from maximality and the quadrant of the Dirac-type CP-violating phase δ can be used to lift this degeneracy.

Mass matrices provide important tools for the investigation of the underlying symmetries and the resulting dynamics. The first step in this direction is the reconstruction of the neutrino mass matrix in the flavor basis. However, the reconstruction results in a large variety of possible structures of mass matrices depending strongly on the mass scale, mass hierarchy and the Majorana phases. However, the relatively weak dependence on some oscillation parameters (θ_{23} and δ) results in the degeneracy of possible neutrino mass matrices since all the parameters are not known at present. The mass matrix for Majorana neutrinos contains nine physical parameters including the three mass eigenvalues, three mixing angles and the three CP-violating phases. The two squared-mass differences (Δm^2_{21} and Δm^2_{32}) and the two mixing angles (θ_{12} and θ_{23}) have been measured in solar, atmospheric and reactor experiments. The third mixing angle θ_{13} and the Dirac-type CP-violating phase δ are expected to be measured in the forthcoming neutrino oscillation experiments. The possible measurement of the effective Majorana mass in neutrinoless double β decay searches will provide an additional constraint on the remaining three neutrino parameters viz. the neutrino mass scale and two Majorana CP violating phases. While the neutrino mass scale will be determined by the direct beta decay searches and cosmological observations, the two Majorana phases will not be uniquely determined even if the absolute neutrino mass

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scale is known. Under the circumstances, it is natural to employ other theoretical inputs for the reconstruction of the neutrino mass matrix. Several proposals have been made in the literature to restrict the form of the neutrino mass matrix and to reduce the number of free parameters which include presence of texture zeros [1, 2, 3, 4], requirement of zero determinant [5, 6] and the zero trace condition [7] to name a few. However, the current neutrino oscillation data are consistent only with a limited number of texture schemes [1, 2, 3, 4]. In particular, the current neutrino oscillation data disallow all neutrino mass matrices with three or more texture zeros in the flavor basis. Out of the fifteen possible neutrino mass matrices with two texture zeros, only seven are compatible with the current neutrino oscillation data. The seven allowed two texture zero mass matrices have been classified into three categories. The two class A matrices of the types A_1 and A_2 give hierarchical neutrino masses. The class B matrices of types B_1, B_2, B_3 and B_4 yield a quasi-degenerate spectrum of neutrino masses. The single class C matrix corresponds to inverted hierarchy of neutrino masses. Furthermore, only the mass matrices of class A and C can accommodate maximal 2-3 mixing and are, hence, favored by the data. Thus, only matrices belonging to class A can accommodate maximal 2-3 mixing and normal hierarchy of neutrino masses.

In the present work, we examine the neutrino mass matrices of class A for which $m_{ee} = 0$ and the neutrino masses are hierarchical. There are two types of mass matrices in class A which are consistent with the neutrino oscillation experiments. For neutrino mass matrices of type A_1

$$m_{ee} = m_{e\mu} = 0 \tag{1}$$

whereas

$$m_{ee} = m_{e\tau} = 0 \tag{2}$$

for neutrino mass matrices of type A_2. The texture zeros within a class were thought to have identical phenomenological consequences [1, 2, 3] leading to the degeneracy of mass matrices of types A_1 and A_2, for example. In the present work, we discuss the ways to lift this degeneracy. We find that the deviation of atmospheric mixing from maximality and the quadrant of the Dirac-type CP-violating phase δ can be used to distinguish the mass matrices of types A_1 and A_2. It is, also, found, that the prospects for the measurement of θ_{13} in neutrino mass matrices of class A are quite optimistic since a definite lower bound on θ_{13} is obtained for this class.

The neutrino mass matrix, in the flavor basis, is given by

$$(m_\nu)_{ij} = (U m_\nu^d U^T)_{ij}; \quad i,j = e,\mu,\tau \tag{3}$$

for Majorana neutrinos where $m_\nu^d = Diag\{m_1, m_2, m_3\}$ is the diagonal neutrino mass matrix and U is the neutrino mixing matrix which is given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix} \tag{4}$$

in PDG representation [8]. Here, $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. For the above parameterization, the ee, $e\mu$ and $e\tau$ elements of the neutrino mass matrix are given by

$$m_{ee} = c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{2i\alpha} + s_{13}^2 m_3 e^{2i\beta}, \tag{5}$$

$$m_{e\mu} = c_{13}\{s_{13}s_{23}e^{i\delta}(e^{2i\beta}m_3 - s_{12}^2 e^{2i\alpha}m_2) - c_{12}c_{23}s_{12}(m_1 - e^{2i\alpha}m_2) - c_{12}^2 s_{13}s_{23}e^{i\delta}m_1\} \quad (6)$$

and

$$m_{e\tau} = c_{13}\{s_{13}c_{23}e^{i\delta}(e^{2i\beta}m_3 - s_{12}^2 e^{2i\alpha}m_2) + c_{12}s_{23}s_{12}(m_1 - e^{2i\alpha}m_2) - c_{12}^2 s_{13}c_{23}e^{i\delta}m_1\}. \quad (7)$$

It will be helpful to note that $m_{e\tau}$ can be obtained from $m_{e\mu}$ by exchanging s_{23} with c_{23} and c_{23} with $-s_{23}$. In other words, the transformation $\theta_{23} \rightarrow \theta_{23} + \frac{\pi}{2}$ transforms $m_{e\mu}$ to $m_{e\tau}$. However, the transformation $\theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}$ followed by the transformation $\delta \rightarrow \delta + \pi$, also, transforms $m_{e\mu}$ to $-m_{e\tau}$. Therefore, if $m_{e\mu}$ vanishes for θ_{23} and δ, then $m_{e\tau}$ vanishes for $\frac{\pi}{2} - \theta_{23}$ and $\delta + \pi$. This symmetry argument clearly shows that the predictions of neutrino mass matrices for types A_1 and A_2 will be identical for all neutrino parameters except θ_{23} and δ. The predictions for θ_{23} and δ can be parameterized as

$$\begin{aligned} A_1 \quad &: \quad \theta_{23} = \frac{\pi}{4} + x, \qquad \delta = y, \\ A_2 \quad &: \quad \theta_{23} = \frac{\pi}{4} - x, \quad \delta = \pi + y. \end{aligned} \quad (8)$$

Since, the two squared-mass differences Δm_{12}^2 and Δm_{23}^2 are known experimentally from the solar, atmospheric and reactor neutrino experiments, we treat m_1 as the free parameter and obtain m_2 and m_3 from m_1 using the following relations:

$$m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}, \quad (9)$$

and

$$m_3 = \sqrt{m_2^2 + \Delta m_{23}^2}. \quad (10)$$

Our present knowledge [9] of the oscillation parameters has been summarized below:

$$\begin{aligned} \Delta m_{12}^2 &= 7.9^{+0.3,1.0}_{-0.3,0.8} \times 10^{-5} eV^2, \\ s_{12}^2 &= 0.31^{+0.02,0.09}_{-0.03,0.07}, \\ \Delta m_{23}^2 &= 2.2^{+0.37,1.1}_{-0.27,0.8} \times 10^{-3} eV^2, \\ s_{23}^2 &= 0.50^{+0.06,0.18}_{-0.05,0.16}, \\ s_{13}^2 &< 0.012(0.046). \end{aligned} \quad (11)$$

Since, there is only an upper bound on θ_{13}, we treat it as an unknown parameter in the beginning and use the CHOOZ bound given above to constrain the allowed parameter space only at the end. Thus, the element m_{ee} is a function of four unknown parameters viz. m_1, α, β and θ_{13} while the elements $m_{e\mu}$ and $m_{e\tau}$ are the functions of five unknown parameters viz. m_1, α, β, δ and θ_{13}.

The condition $m_{ee} = 0$ has, already, been examined in detail [10]. For $\theta_{13} = 0$, m_{ee} vanishes for $\alpha = 90^0$ and $m_1 = s_{12}^2\sqrt{\frac{\Delta m_{12}^2}{\cos 2\theta_{12}}}$. For non-zero θ_{13}, the element m_{ee} can vanish only in the normal hierarchy. The real and imaginary parts of m_{ee} are separately zero if

$$s_{13}^2 = \frac{c_{12}^2 m_1 + s_{12}^2 m_2 \cos 2\alpha}{\mu_1} \quad (12)$$

and

$$s_{13}^2 = \frac{s_{12}^2 m_2 \sin 2\alpha}{\mu_2} \tag{13}$$

where, μ_1 and μ_2 are given by

$$\mu_1 = m_1 c_{12}^2 + m_2 s_{12}^2 \cos 2\alpha - m_3 \cos 2\beta, \tag{14}$$

$$\mu_2 = m_2 s_{12}^2 \sin 2\alpha - m_3 \sin 2\beta. \tag{15}$$

The two values of s_{13}^2 given in Eq. (12) and Eq. (13) are equal if

$$\sin 2\beta = -\frac{s_{12}^2 m_2 \sin 2\alpha}{M} \tag{16}$$

and

$$\cos 2\beta = -\frac{c_{12}^2 m_1 + s_{12}^2 m_2 \cos 2\alpha}{M} \tag{17}$$

and the value of s_{13}^2 is given by

$$s_{13}^2 = \frac{M}{M + m_3} \tag{18}$$

where

$$M = \sqrt{m_1^2 c_{12}^4 + m_2^2 s_{12}^4 + 2 m_1 m_2 c_{12}^2 s_{12}^2 \cos 2\alpha}. \tag{19}$$

Using Eqs. (16) and (17), the parameters μ_1 and μ_2 defined in Eqs. (14) and (15) can be written in terms of a single Majorana phase α or β. We express them as functions of β:

$$\begin{aligned} \mu_1 &= -(M + m_3) \cos 2\beta, \\ \mu_2 &= -(M + m_3) \sin 2\beta. \end{aligned} \tag{20}$$

Next, we examine the conditions $m_{e\mu} = 0$ and $m_{e\tau} = 0$. For vanishing θ_{13}, $m_{e\mu}$ or $m_{e\tau}$ can only be zero if $m_1 = m_2$ and $\alpha = 90^0$ which implies a lower bound on θ_{13} [4]. Therefore, the angle θ_{13} will be bounded from below for neutrino mass matrices of class A.

The element $m_{e\mu}$ vanishes if $Re(m_{e\mu}) = 0$ and $Im(m_{e\mu}) = 0$ which yields the following two relationships for s_{13}:

$$s_{13} = -\frac{c_{12} c_{23} s_{12}}{s_{23}} \frac{m_1 - m_2 \cos 2\alpha}{\mu_1 \cos\delta - \mu_2 \sin\delta} \tag{21}$$

and

$$s_{13} = \frac{c_{12} c_{23} s_{12}}{s_{23}} \frac{m_2 \sin 2\alpha}{\mu_1 \sin\delta + \mu_2 \cos\delta} \tag{22}$$

where the parameters μ_1 and μ_2 are given by Eqs. (14) and (15). The two values of s_{13} will be equal if

$$\begin{aligned} \sin\delta &= \frac{\mu_1 m_2 \sin 2\alpha - \mu_2 (m_2 \cos 2\alpha - m_1)}{\mu_3 \sqrt{\mu_1^2 + \mu_2^2}}, \\ \cos\delta &= \frac{\mu_2 m_2 \sin 2\alpha + \mu_1 (m_2 \cos 2\alpha - m_1)}{\mu_3 \sqrt{\mu_1^2 + \mu_2^2}} \end{aligned} \tag{23}$$

and s_{13} is given by

$$s_{13} = \frac{c_{12}c_{23}s_{12}}{s_{23}}\frac{\mu_3}{\sqrt{\mu_1^2+\mu_2^2}} \tag{24}$$

where

$$\mu_3 = \sqrt{m_1^2+m_2^2-2m_1m_2\cos 2\alpha}. \tag{25}$$

Therefore, δ and s_{13} become functions of m_1, α and β for vanishing $m_{e\mu}$. Similar expressions for δ and s_{13} can be obtained for vanishing $m_{e\tau}$ from Eqs. (23) and (24) by applying the transformations $\theta_{23} \longrightarrow \frac{\pi}{2} - \theta_{23}$ and $\delta \longrightarrow \delta + \pi$. Therefore, $m_{e\tau}$ vanishes if

$$\begin{aligned}
\sin\delta &= -\frac{\mu_1 m_2 \sin 2\alpha - \mu_2\left(m_2\cos 2\alpha - m_1\right)}{\mu_3\sqrt{\mu_1^2+\mu_2^2}}, \\
\cos\delta &= -\frac{\mu_2 m_2 \sin 2\alpha + \mu_1\left(m_2\cos 2\alpha - m_1\right)}{\mu_3\sqrt{\mu_1^2+\mu_2^2}}
\end{aligned} \tag{26}$$

and s_{13} is, now, given by

$$s_{13} = \frac{c_{12}s_{23}s_{12}}{c_{23}}\frac{\mu_3}{\sqrt{\mu_1^2+\mu_2^2}}. \tag{27}$$

The transformation $\delta \longrightarrow \delta + \pi$ keeps $\tan\delta$ unchanged. Equations (26) and (27) can, also, be derived directly in the same way as we derived Eqs. (23) and (24).

Now, we examine the implications of vanishing m_{ee} and $m_{e\mu}$ for the neutrino mass matrices of type A_1. Since, θ_{13} must vanish for $\alpha = 90^0$ if $m_{ee} = 0$, and a vanishing θ_{13} is not allowed by the condition $m_{e\mu} = 0$, the points $\alpha = 90^0$ and $\theta_{13} = 0$ are not allowed for the neutrino mass matrices of type A_1.

Substituting the values of μ_1 and μ_2 from Eqs. (20) in Eqs. (23), we obtain

$$\begin{aligned}
\sin\delta &= -\frac{m_2\sin 2(\alpha-\beta) + m_1\sin 2\beta}{\mu_3}, \\
\cos\delta &= -\frac{m_2\cos 2(\alpha-\beta) - m_1\cos 2\beta}{\mu_3}.
\end{aligned} \tag{28}$$

Eliminating β from the Eqs. (28) by using the Eqs. (16) and (17), we obtain

$$\sin\delta = \frac{m_1 m_2 \sin 2\alpha}{M\mu_3} \tag{29}$$

and

$$\cos\delta = \frac{s_{12}^2 m_2^2 - c_{12}^2 m_1^2 + m_1 m_2 (c_{12}^2 - s_{12}^2)\cos 2\alpha}{M\mu_3}. \tag{30}$$

The Dirac-type CP-violating phase δ can, also, be written in terms of β as

$$\sin\delta = -\frac{m_1 \sin 2\beta}{s_{12}^2\mu_3}. \tag{31}$$

The value $\alpha = 90^0$ corresponds to $\beta = 0^0$ and $\delta = 0^0$. Since, $\alpha = 90^0$ is not allowed, $\beta = 0^0$ and $\delta = 0^0$ are, also, not allowed. Therefore, two texture zero mass matrices of class A are necessarily CP violating. From Eq. (24), we obtain

$$s_{13} = \frac{c_{12}c_{23}s_{12}}{s_{23}} \frac{\mu_3}{M + \mu_3}. \tag{32}$$

For the simultaneous existence of texture zeros at the ee and $\mu\mu$ entries, the two relations for s_{13} calculated from the conditions $m_{ee} = 0$ and $m_{e\mu} = 0$ should be consistent with each other. Equations (18) and (34) give the same value of s_{13} for

$$M(M + m_3)\tan^2\theta_{23} = \mu_3^2 s_{12}^2 c_{12}^2 \tag{33}$$

and the value of s_{13} is given by

$$s_{13} = \frac{s_{23}^2}{c_{12}^2 c_{23}^2 s_{12}^2} \frac{M}{\mu_3}. \tag{34}$$

Eq. (33) can be solved to obtain α as a function of m_1 for the neutrino mass matrices of type A_1. With this value of α, one can calculate δ and θ_{13} from Eqs. (29), (30) and (34). Eq. (34) can be used to obtain a lower bound on θ_{13}:

$$s_{13} > \frac{s_{23}^2}{c_{12}^2 c_{23}^2 s_{12}^2} \frac{|m_2 s_{12}^2 - m_1 c_{12}^2|}{m_1 + m_2}. \tag{35}$$

Similarly, one can show that for neutrino mass matrices of type A_2, δ and s_{13} are given by

$$\sin\delta = \frac{m_1 \sin 2\beta}{s_{12}^2 \mu_3} \tag{36}$$

and

$$s_{13} = \frac{c_{23}^2}{c_{12}^2 s_{23}^2 s_{12}^2} \frac{M}{\mu_3}. \tag{37}$$

and the condition for the simultaneous existence of two texture zeros is

$$M(M + m_3) = \mu_3^2 s_{12}^2 c_{12}^2 \tan^2\theta_{23}. \tag{38}$$

We note the reciprocity between the values of $\tan^2\theta_{23}$ in Eqs. (33) and (38) for mass matrices of types A_1 and A_2, respectively. This reciprocity can be, gainfully, used to examine deviations from maximality for the atmospheric mixing angle for matrices of types A_1 and A_2 and, hence, to distinguish between them.

At $\alpha = 90^0$, we have $m_1 = s_{12}^2\sqrt{\frac{\Delta m^2}{\cos 2\theta_{12}}}$ and $\delta + 2\beta = n\pi$ where n is even for A_1 and odd for A_2 type neutrino mass matrices. However, $\alpha = 90^0$ is not allowed, the relation between β and δ remains approximately valid even for small deviations about $\alpha = 90^0$.

The numerical analysis for the neutrino mass matrices of type A_1 is done in the following manner. The oscillation parameters Δm_{12}^2, Δm_{23}^2, s_{12}^2 and s_{23}^2 are varied within their experimental ranges given in Eqs. (11) and α is calculated as a function of m_1 by using Eq. (33). For these (m_1,α) values, β is calculated from Eqs. (16) and (17), δ from Eqs. (29) and (30)

C.L.	$m_1(10^{-3}eV)$	α(deg)	β(deg)	θ_{13}(deg)	J
1	2.8 - 4.2	83.4 - 96.6	-59.4 - 59.4	5.6 - 6.3	-0.011 - 0.011
2	2.0 - 10.0	76.4 - 103.6	-90 - 90	4.6 - 9.8	-0.021 - 0.021
3	1.6 - 16.2	71.2 - 108.8	-90 - 90	3.5 - 12.4	-0.046 - 0.046

Table 1: The predictions for neutrino mass matrices of class A.

and θ_{13} from Eq. (34). The CHOOZ bound on θ_{13} is used to constrain m_1 and, hence, all the other neutrino parameters. The neutrino mass matrices of type A_2 can be studied in an analogous manner. The results have been summarized in Table 1 at various confidence levels for m_1, α, β, θ_{13} and Jarlskog rephasing invariant quantity [11]

$$J = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin\delta. \tag{39}$$

These quantities are the same for mass matrices of types A_1 and A_2. It can bee seen from Table 1 that the 3 σ lower bound on θ_{13} is 3.5^0. The range for the Majorana-type CP-violating phase β at 1 σ C.L. is found to be -59.4^0 - 59.4^0. However, if the neutrino oscillation parameters are allowed to vary beyond their present 1.2 σ C.L. ranges, the full range for β (-90^0 - 90^0) is allowed. As noted earlier, the neutrino mass matrices of type A_1 and A_2 differ in their predictions for δ and θ_{23}. At one standard deviation, the allowed range of δ is (-110.8^0 - 110.8^0) for type A_1 and (69.2^0 - 290.8^0) for type A_2 and the allowed range of θ_{23} is (44.0^0 - 48.2^0) for type A_1 and (41.8^0 - 46.0^0) for type A_2 [Table 2]. Just like β, no constraint on δ is obtained above 1.2 σ C.L. The neutrino mass matrices of types A_1 and A_2 have some overlap in their predictions regarding δ and θ_{23} at one standard deviation. Therefore, a precise measurement of the oscillation parameters is necessary to distinguish the neutrino mass matrices of types A_1 and A_2.

In Fig. 1, we depict the allowed values of m_1, α and β as the correlation plots at one standard deviation. The upper panel shows α as a function of m_1 and the lower panel shows β as a function of α. The large spread in the (m_1,α) plot is due to the errors in the neutrino oscillation parameters. However, there is a very strong correlation between α and β as indicated by Eqs. (16) and (17). As seen earlier, the points $\alpha = 90^0$ and $\beta = 0^0$ are excluded in Fig. 1. The two Majorana phases are strongly correlated with each other. Such a correlation between the Majorana phases was noted earlier by Xing [3]. However, full ranges (-90^0 - 90^0) are allowed for the two Majorana phases in that analysis [3]. In contrast, we obtain a very narrow range for the Majorana phase α around 90^0 [Table 1]. Similarly, the Majorana phase β is, also, constrained if the oscillation parameters are limited to their 1 σ ranges. In Fig. 2, we depict the correlation plots of α and β with δ for matrices

1 σ predictions	δ	θ_{23}
A_1	-110.8^0 - 110.8^0	44.0^0 - 48.2^0
A_2	69.2^0 - 290.8^0	41.8^0 - 46.0^0

Table 2: The predictions for δ and θ_{23}.

of types A_1 (left panel) and A_2 (right panel). We, also, show the correlation plots of δ and θ_{23} with one another as well as with θ_{13} in Fig. 2. The Dirac-type CP-violating phase δ is strongly correlated with the Majorana-type CP-violating phases α and β [Cf. Eqs. (29) and (31)]. It can be seen from (α,δ) and (β,δ) correlation plots that there are small deviations in the values of α around 90^0 and the correlation between β and δ is almost linear. The fact that $\delta + 2\beta \simeq 0^0$ for A_1 type mass matrices and $\delta + 2\beta \simeq 180^0$ for A_2 type mass matrices is apparent from the (β,δ) correlation plots given in the left and right panels in Fig. 2, respectively. The (β,δ) correlation was noted earlier by Xing [3] in its approximate form in a different parameterization. The (δ, θ_{23}) plots, clearly, illustrate the point that the neutrino mass matrices of types A_1 and A_2 have different predictions for these variables [Table 2] and only a limited region is allowed on the (δ, θ_{23}) plane. This is contrary to the analysis done by Xing [3] where no constraints on δ and θ_{23} have been obtained. In fact, this feature is crucial for distinguishing mass matrices of types A_1 and A_2 which were found to be degenerate in the earlier analyses [1, 2, 3]. The constraints on δ and θ_{23} are very sensitive to the values of θ_{13}. For the values of θ_{13} smaller than 1σ CHOOZ bound, the constraints on δ and θ_{23} become stronger which can be seen from the (θ_{13}, θ_{23}) and (θ_{13},δ) plots. For example, if $\theta_{13} > 6^0$, then $\theta_{23} > 46^0$ (above maximal) for type A_1 and $\theta_{23} < 44^0$ (below maximal) for type A_2. It can, also, be seen from the (θ_{13}, θ_{23}) correlation plot in Fig. 2 that the deviation of θ_{23} from maximality is larger for smaller values of θ_{13}. Therefore, if future experiments measure θ_{13} below its present 1σ bound, the neutrino mass matrices of types A_1 and A_2 will have different predictions for δ and θ_{23} with no overlap. However, it would be difficult to differentiate between matrices of types A_1 and A_2 if θ_{13} is found to be above its present 1σ range. As noted earlier, different quadrants for δ are selected for neutrino mass matrices of types A_1 and A_2. It can, also, be seen in Fig. 2 that the points $\delta = 0, \pi$ are ruled out implying that neutrino mass matrices of class A are necessarily CP-violating. This feature is, also, apparent in Fig. 3 where we have plotted J as a function of m_1 and in Fig. 4 (identical for A_1 and A_2) which depicts J as a function of δ for types A_1 (left panel) and A_2(right panel). The symmetry argument summarized in Eq. (8) is consistent with the numerical results presented in Fig. 2.

If neutrinoless double beta decay searches give positive results and m_{ee} is measured experimentally or the atmospheric/reactor neutrino oscillation experiments confirm inverted hierarchy, the neutrino mass matrices of class A will be ruled out. A generic prediction of this class of models is a lower bound on θ_{13} [Table 1]. If the forthcoming neutrino oscillation experiments measure θ_{13} below 3.5^0, the neutrino mass matrices of class A will, again, be ruled out. The forthcoming neutrino experiments will aim at measuring the Dirac type CP violating phase δ and the deviations of the atmospheric mixing angle from maximality [12]. The results of these experiments will fall in one of the following four categories of phenomenological interest:

1. $\theta_{23} > 45^0$ and $-90^0 < \delta < 90^0$,
2. $\theta_{23} < 45^0$ and $90^0 < \delta < 270^0$,
3. $\theta_{23} < 45^0$ and $-90^0 < \delta < 90^0$,
4. $\theta_{23} > 45^0$ and $90^0 < \delta < 270^0$.

If the experiments confirm the first possibility, the neutrino mass matrices of type A_1 may explain the data. If the experiments confirm the second possibility, the mass matrices of type A_2 may be allowed. In case, the experiments select the third or fourth possibility, the neutrino mass matrices of class A will be ruled out. If the neutrino mass matrices of class A are confirmed experimentally, this would correlate the CP-violation induced by the Dirac phase δ with the CP-violation induced by Majorana phases in a definite way.

In conclusion, the degeneracy between A_1 and A_2 type neutrino mass matrices has been examined in detail. In the earlier analyses [1, 2, 3], textures within a class were found to be experimentally indistinguishable. However, our analysis shows that the deviations of the atmospheric mixing from the maximality and the quadrant of the Dirac-type CP-violating phase δ can be used to lift this degeneracy between mass matrices of types A_1 and A_2. Contrary to the results of earlier analyses [3], which leave the Dirac phase completely unconstrained, we have obtained interesting constraints not only for the Dirac phase but, also, for the Majorana-type CP-violating phases. Our analysis, also, shows that the prospects for the measurement of θ_{13} for neutrino mass matrices of class A are quite optimistic. Moreover, the neutrino mass matrices of class A connect the deviation of θ_{23} from maximality with the quadrant of δ. Similarly, the CP-violation observed in the neutrino oscillation experiments is linked with the CP-violation induced by Majorana phases.

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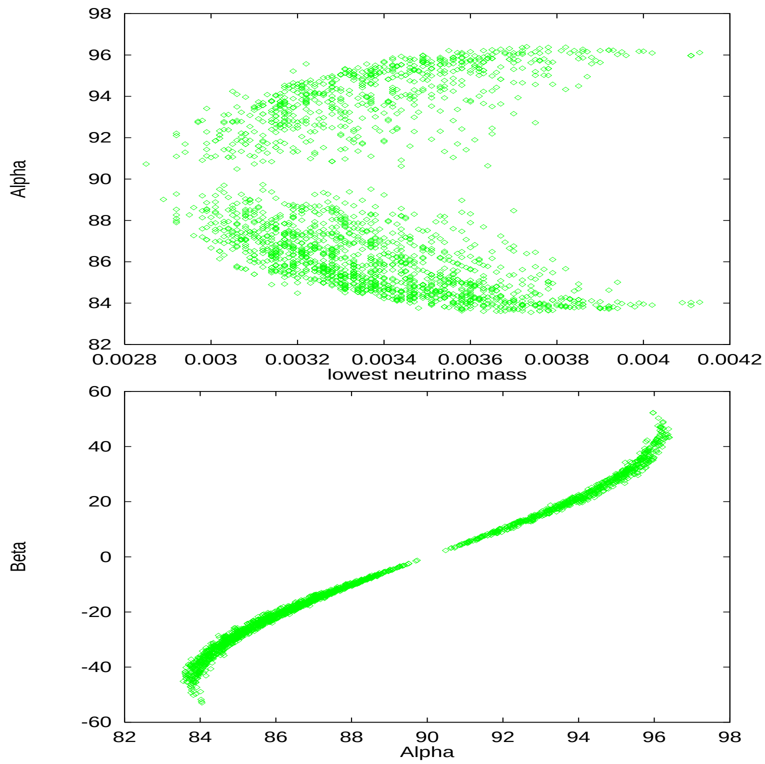

Figure 1: Correlation plots for neutrino mass matrices of class A at one standard deviation.

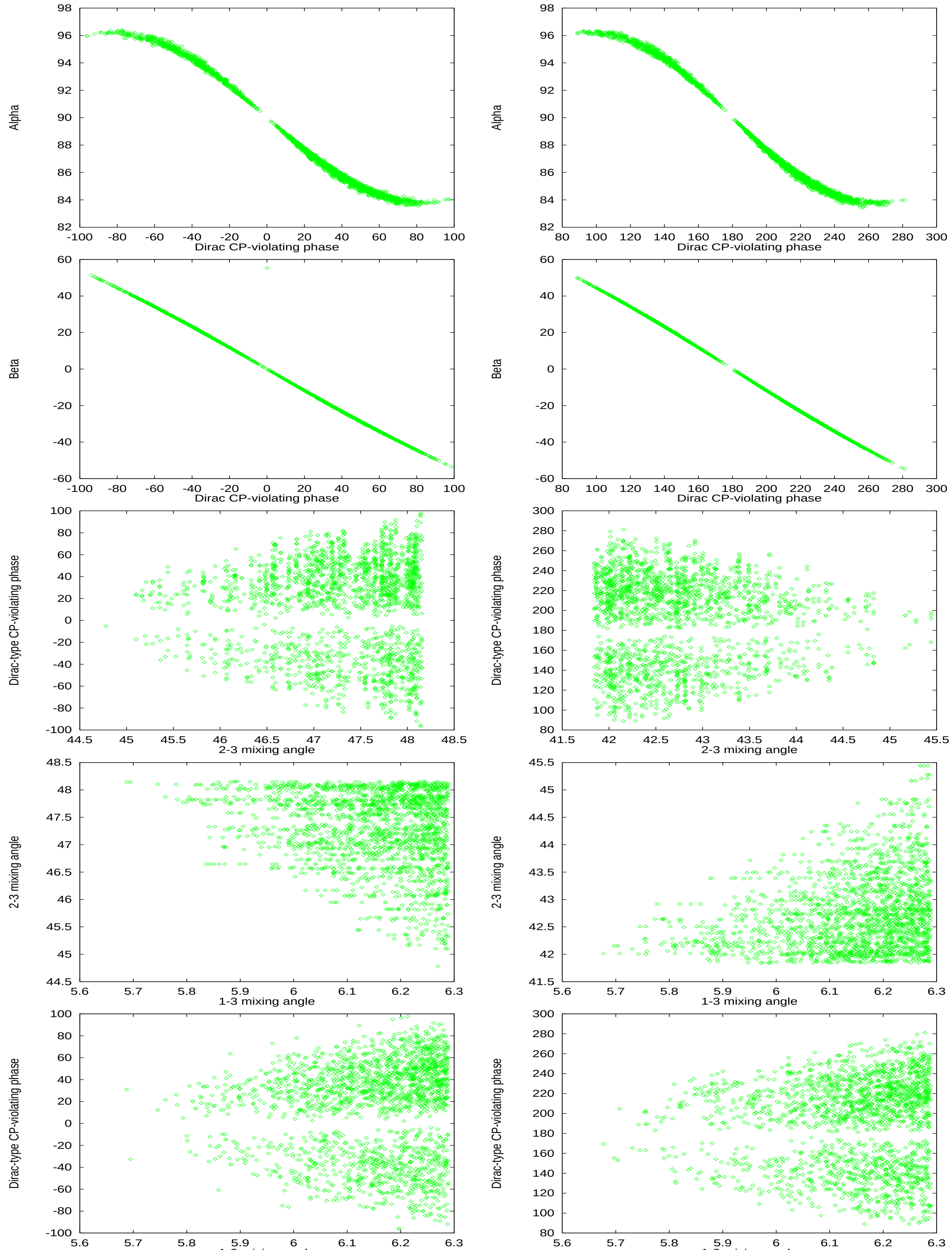

Figure 2: Correlation plots for neutrino mass matrices of types A_1 (left panel) and A_2 (right panel).

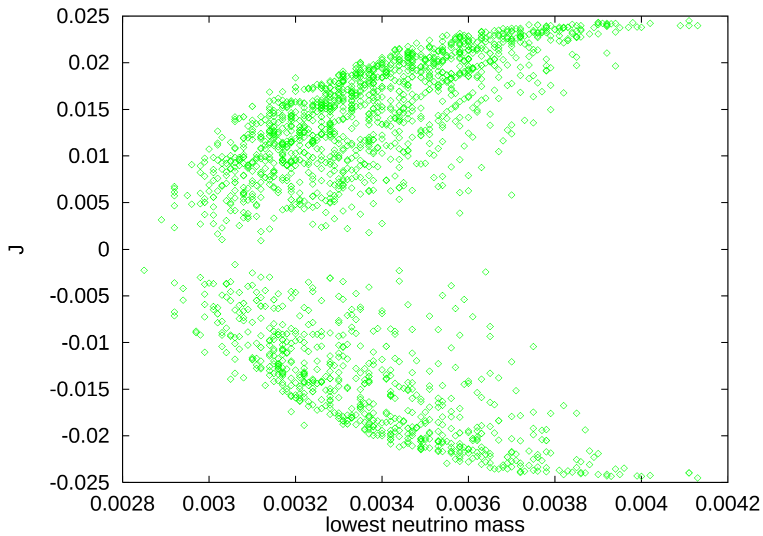

Figure 3: J as a function of m_1 for the neutrino mass matrices of class A.

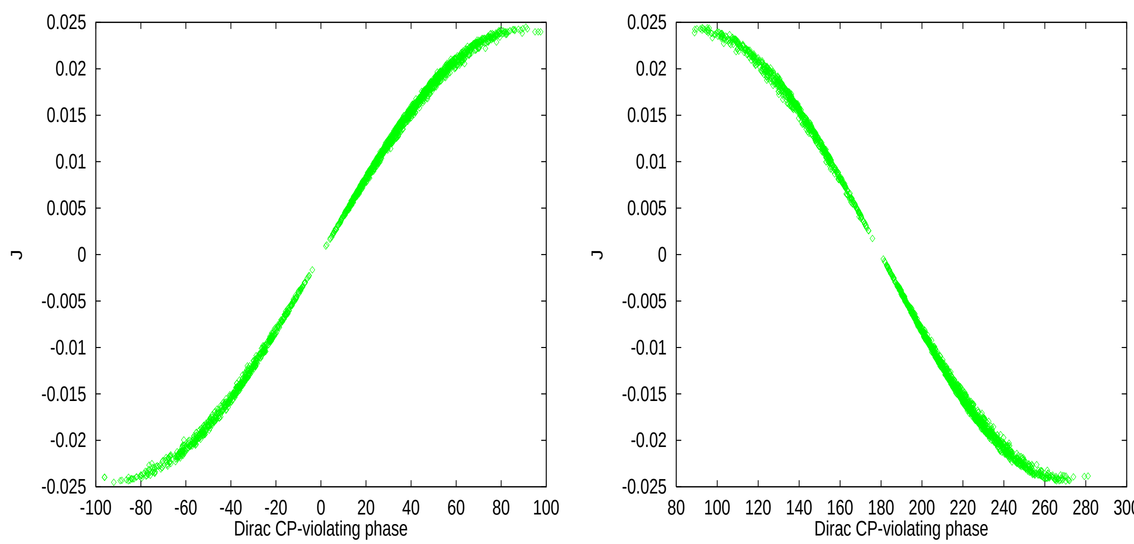

Figure 4: J as a function of δ for the neutrino mass matrices of types A_1 (left panel) and A_2 (right panel).